Exercise 30

Differentiate f and find the domain of f.

$$f(x) = \ln \ln \ln x$$

Solution

Recognize that only the logarithm of a positive number can be taken.

$$\begin{aligned}
\ln \ln x > 0 \quad &\text{and} \quad \ln x > 0 \quad \text{and} \quad x > 0 \\
\ln x > e^0 \quad &\text{and} \quad x > e^0 \quad \text{and} \quad x > 0 \\
\ln x > 1 \quad &\text{and} \quad x > 1 \quad \text{and} \quad x > 0 \\
x > e^1 \quad &\text{and} \quad x > 1 \quad \text{and} \quad x > 0 \\
x > e \quad &\text{and} \quad x > 1 \quad \text{and} \quad x > 0
\end{aligned}$$

Therefore, the domain of the function is

$$(e, \infty).$$

Take the derivative of the function with respect to x by using the chain rule repeatedly.

$$\begin{aligned}
f'(x) &= \frac{d}{dx}\{\ln[\ln(\ln x)]\} \\
&= \frac{1}{\ln(\ln x)} \cdot \frac{d}{dx}[\ln(\ln x)] \\
&= \frac{1}{\ln(\ln x)} \cdot \left[\frac{1}{\ln x} \cdot \frac{d}{dx}(\ln x)\right] \\
&= \frac{1}{\ln(\ln x)} \cdot \left[\frac{1}{\ln x} \cdot \left(\frac{1}{x}\right)\right] \\
&= \frac{1}{x(\ln x)\ln(\ln x)}
\end{aligned}$$